MASS: A Complexity-Optimal Solution for Product Kernel Density Visualization

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Overview of Kernel Density Visualization (KDV)

CHALLENGE!

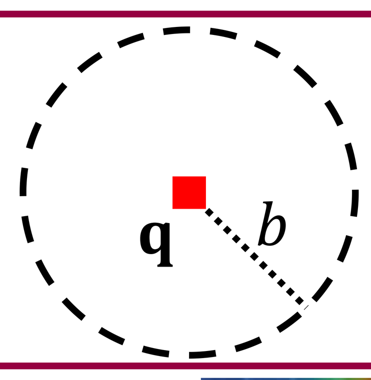
Method	Time Complexity
SCAN	$O(XYn)$
SLAM	$O(X(Y+n))/O(Y(X+n))$

Fastest! But still dominated by Xn/Yn.

- Slow for **large datasets** or **high resolutions**
- Hard to meet **real-time** interaction (< 0.5 s)

$d(\mathbf{q},\mathbf{p}) \le b$

circular search space $\mathbb{S}(\mathbf{q})$

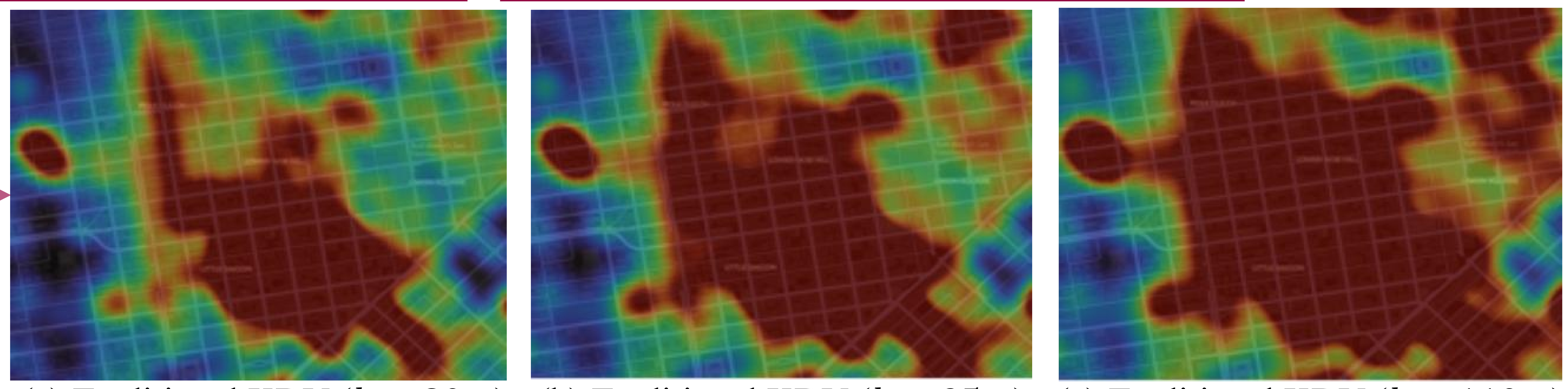


Traditional KDV

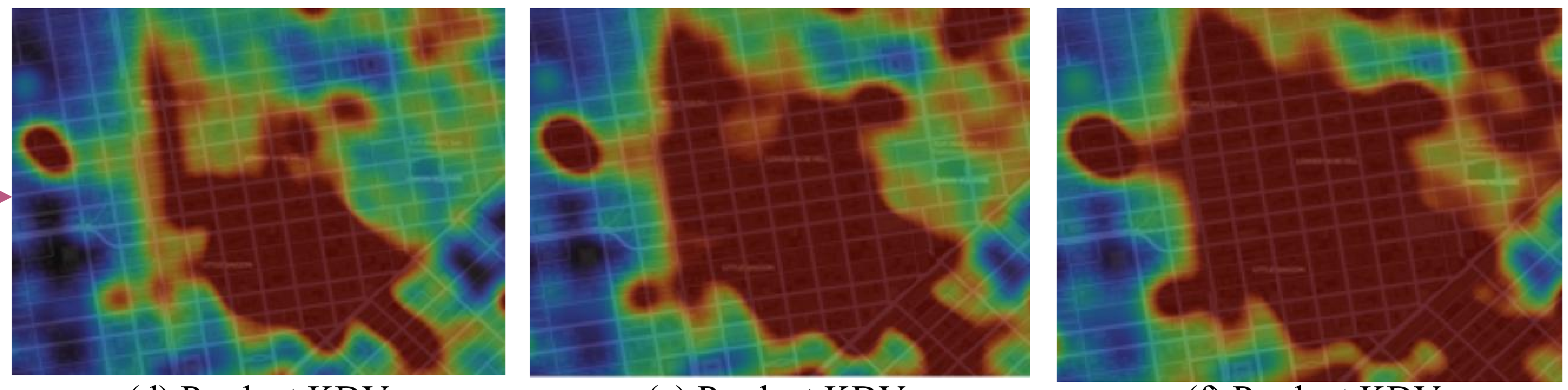
$$\mathcal{D}_P^T(\mathbf{q}) = \sum_{\mathbf{p}\in P} w \cdot \begin{cases} 1-\frac{1}{b^2}d(\mathbf{q},\mathbf{p})^2, & d(\mathbf{q},\mathbf{p}) \le b \\ 0, & \text{otherwise} \end{cases}$$

Product KDV

$$\mathcal{D}_P(\mathbf{q}) = \sum_{\mathbf{p}\in P} w \cdot \begin{cases} 1-\frac{1}{{b_x}^2}d(\mathbf{q}_x,\mathbf{p}_x)^2, & d(\mathbf{q}_x,\mathbf{p}_x) \le b_x \\ 0, & \text{otherwise} \end{cases} \cdot \begin{cases} 1-\frac{1}{{b_y}^2}d(\mathbf{q}_y,\mathbf{p}_y)^2, & d(\mathbf{q}_y,\mathbf{p}_y) \le b_y \\ 0, & \text{otherwise} \end{cases}$$



(a) Traditional KDV ($b = 80m$) (b) Traditional KDV ($b = 95m$) (c) Traditional KDV ($b = 110m$)



(d) Product KDV ($b_x = 73m$ and $b_y = 73m$) (e) Product KDV ($b_x = 88m$ and $b_y = 88m$) (f) Product KDV ($b_x = 103m$ and $b_y = 103m$)

Generating traditional and product KDVs for the San Francisco 311-call location dataset.

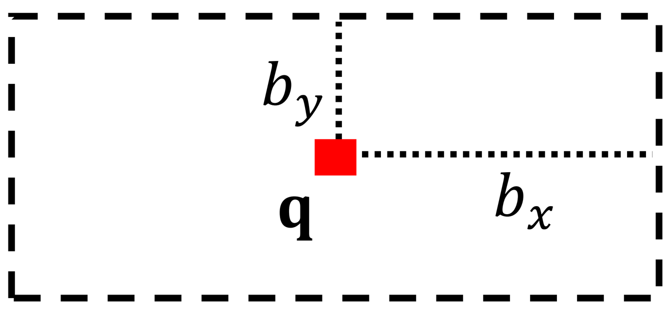
OUR SOLUTIONS

Method	Time Complexity
MASS_{CR}	$O((XY+n)\log(XY))$
MASS_{OPT}	$O(XY+n)$

Optimal Complexity!

$d(\mathbf{q}_x,\mathbf{p}_x) \le b_x$, $d(\mathbf{q}_y,\mathbf{p}_y) \le b_y$

rectangular search space $\mathbb{S}(\mathbf{q})$



Core ideas of Our Solutions (MASS_{CR} and MASS_{OPT})

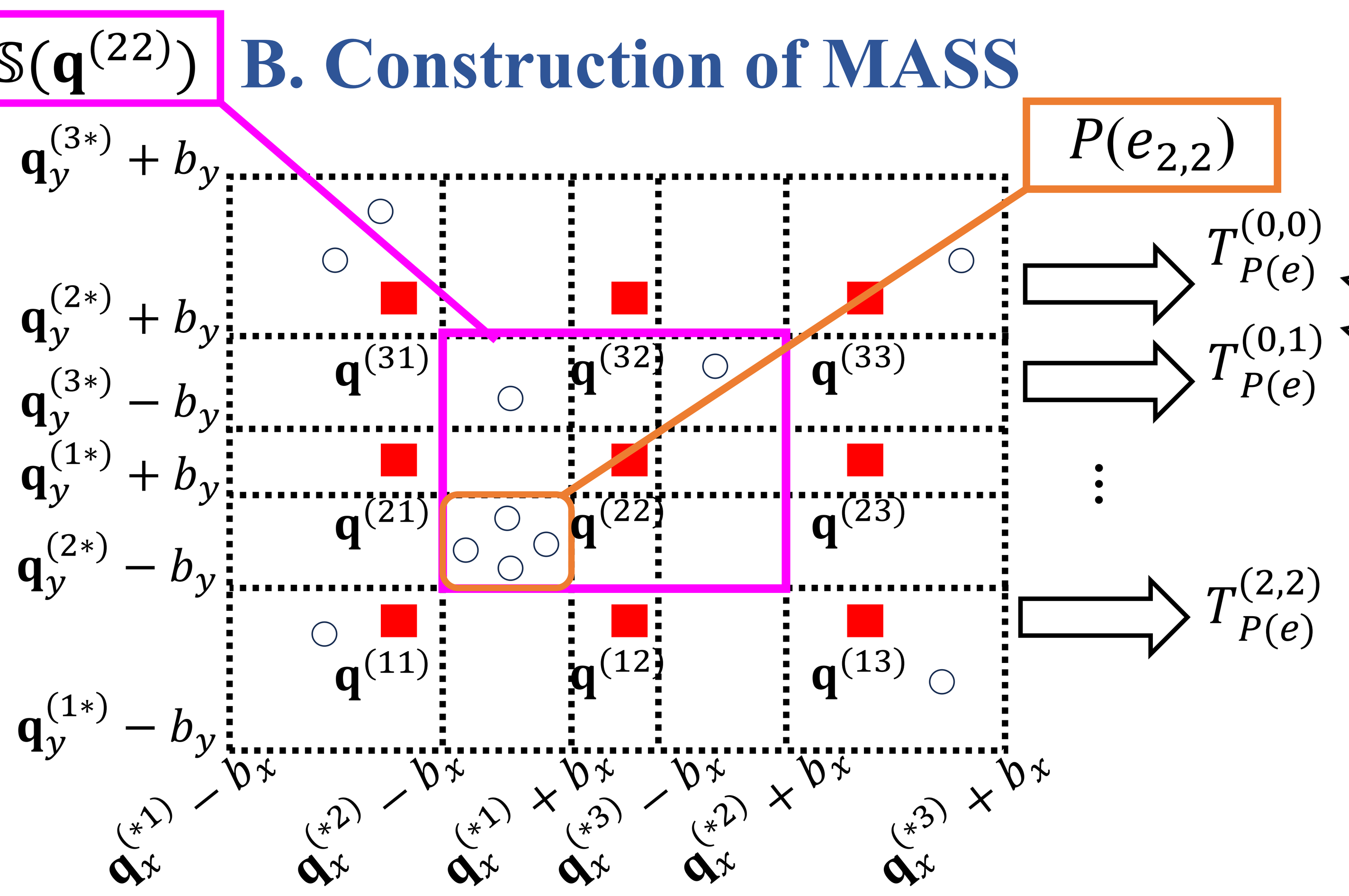
A. Representation of $\mathcal{D}_P(\mathbf{q})$

$$\mathcal{D}_P(\mathbf{q}) = \sum_{\mathbf{p}\in\mathbb{S}(\mathbf{q})} w \cdot \left(1-\frac{1}{b_x^2}d(\mathbf{q}_x,\mathbf{p}_x)^2\right)\cdot\left(1-\frac{1}{b_y^2}d(\mathbf{q}_y,\mathbf{p}_y)^2\right)$$

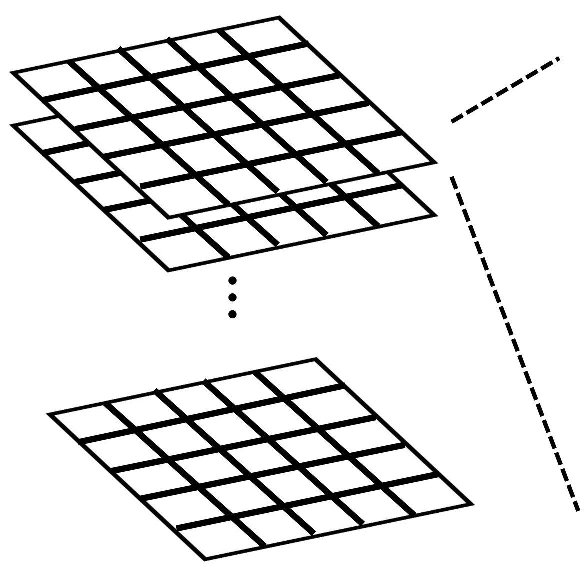
$$= \left(1-\frac{\mathbf{q}_x^2}{b_x^2}\right)\left(1-\frac{\mathbf{q}_y^2}{b_y^2}\right)\boxed{T_{\mathbb{S}(\mathbf{q})}^{(0,0)}} + \frac{2\mathbf{q}_x}{b_x^2}\left(1-\frac{\mathbf{q}_y^2}{b_y^2}\right)\boxed{T_{\mathbb{S}(\mathbf{q})}^{(1,0)}} - \frac{1}{b_x^2}\left(1-\frac{\mathbf{q}_y^2}{b_y^2}\right)\boxed{T_{\mathbb{S}(\mathbf{q})}^{(2,0)}} + \frac{2\mathbf{q}_y}{b_y^2}\left(1-\frac{\mathbf{q}_x^2}{b_x^2}\right)\boxed{T_{\mathbb{S}(\mathbf{q})}^{(0,1)}} + \frac{4\mathbf{q}_x\mathbf{q}_y}{b_x^2b_y^2}\boxed{T_{\mathbb{S}(\mathbf{q})}^{(1,1)}} - \frac{2\mathbf{q}_y}{b_x^2b_y^2}\boxed{T_{\mathbb{S}(\mathbf{q})}^{(2,1)}} - \frac{1}{b_y^2}\left(1-\frac{\mathbf{q}_x^2}{b_x^2}\right)\boxed{T_{\mathbb{S}(\mathbf{q})}^{(0,2)}} - \frac{2\mathbf{q}_x}{b_x^2b_y^2}\boxed{T_{\mathbb{S}(\mathbf{q})}^{(1,2)}} + \frac{1}{b_x^2b_y^2}\boxed{T_{\mathbb{S}(\mathbf{q})}^{(2,2)}}$$

$$T_{S(\mathbf{q})}^{(i,j)} = \sum_{\boxed{\mathbf{p}\in\mathbb{S}(\mathbf{q})}} w\cdot\mathbf{p}_x^i\mathbf{p}_y^j = \sum_{\boxed{\mathbf{p}\in P(e)\subseteq\mathbb{S}(\mathbf{q})}} w\cdot p_x^i p_y^j = \sum_{\boxed{P(e)\subseteq\mathbb{S}(\mathbf{q})}} T_{P(e)}^{(i,j)}$$

B. Construction of MASS



C. Augmentation for MASS



5	2	0	0	0	1
4	0	1	0	1	0
3	0	0	0	0	0
2	0	4	0	0	0
1	1	0	0	0	1
	1	2	3	4	5

$T_{P(e)}^{(0,0)}$ (with $w = 1$)

(0, 0)-MASS

D. Prefix Matrices for MASS

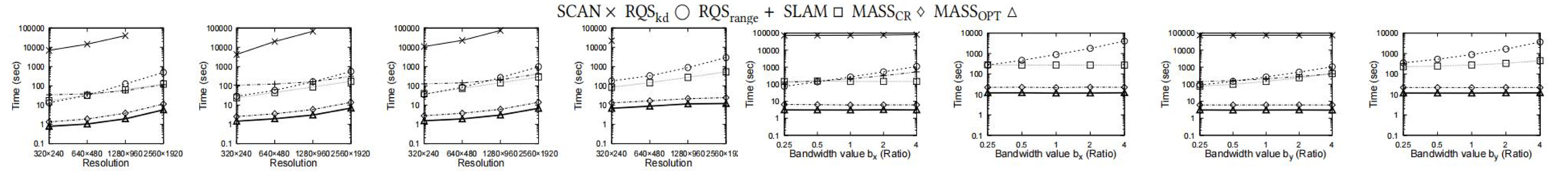
5	3	8	8	9	11
4	1	6	6	7	8
3	1	5	5	5	6
2	1	5	5	5	6
1	1	1	1	1	2
	1	2	3	4	5

Prefix matrix for $T_{P(e)}^{(0,0)}$

$$T_{\mathbb{S}(\mathbf{q}^{(22)})}^{(0,0)} = 7-1-1+1 = 6$$

Experimental Results

SCAN × RQS_{kd} ○ $\text{RQS}_{\text{range}}$ + SLAM □ MASS_{CR} ◇ MASS_{OPT} △



(a) London (b) San Francisco (c) Chicago (d) Manhattan (a) Chicago (varying b_x) (b) Manhattan (varying b_x) (c) Chicago (varying b_y) (d) Manhattan (varying b_y)