DNA: A Distribution-and-Aggregation Solution for Spatiotemporal K-function-based Analysis

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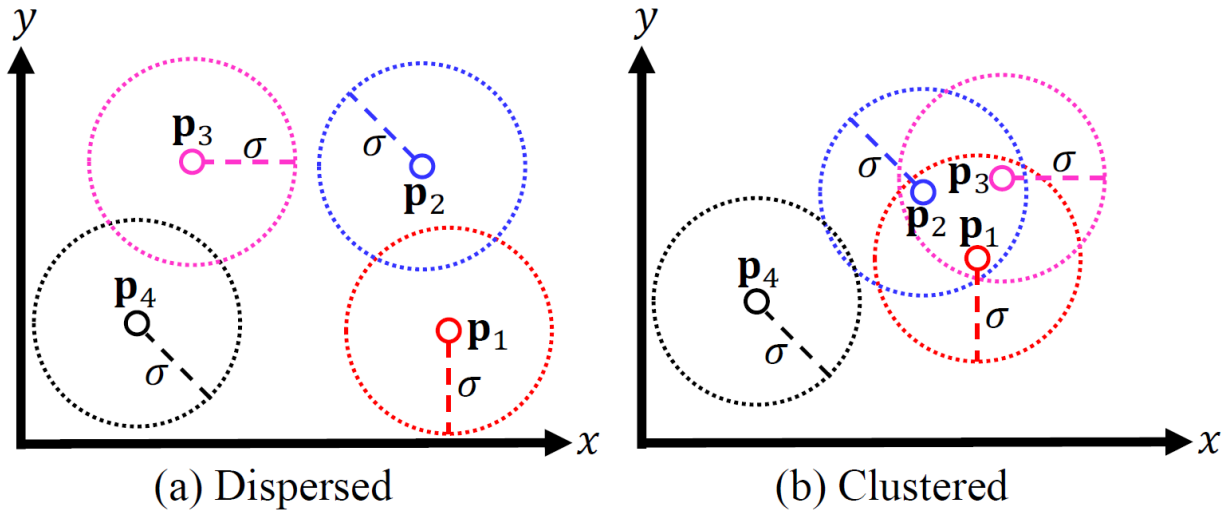
What is K-function?

- Count the number of data points that are within the spatial threshold σ from each data point in a dataset.



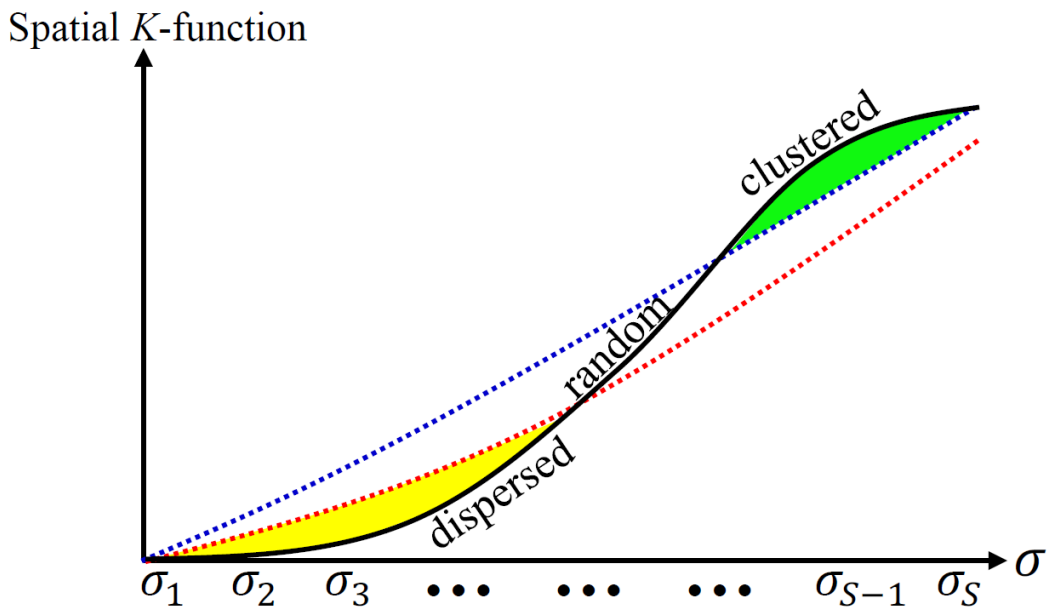
(a) Dispersed (b) Clustered

- If this number is large, the dataset tends to be clustered. Otherwise, the dataset tends to be dispersed.

What is K-function Plot?

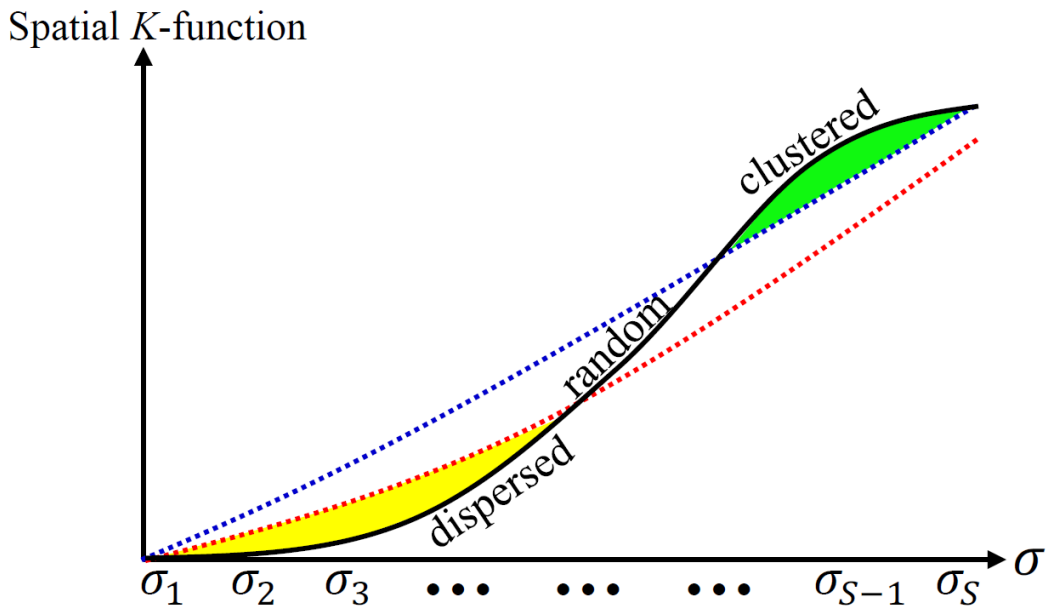
- Obtain the K-function values (with respect to different thresholds) for the original dataset (black curve).

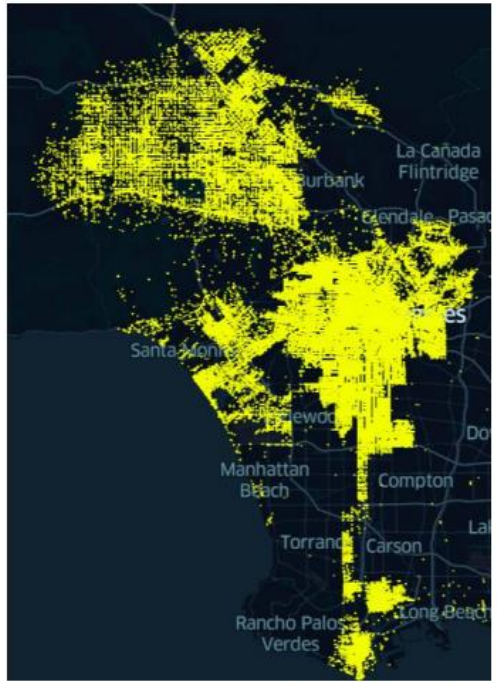
- Generate L random datasets and obtain the minimum and maximum K-function values for each threshold (the red dotted curve and blue dotted curve, respectively).



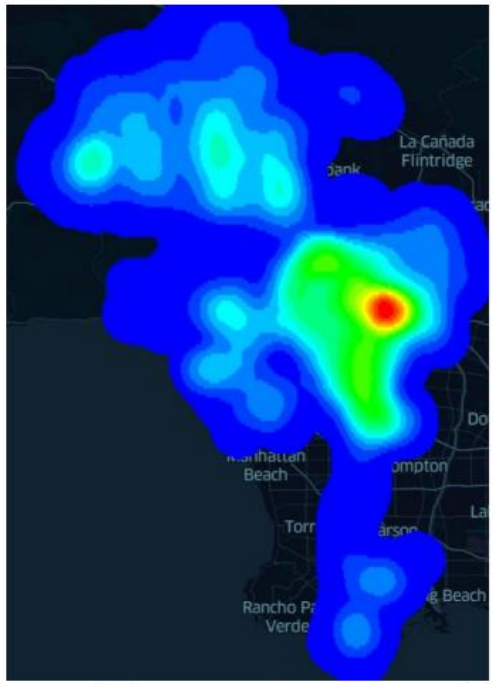
Why K-function Plot?

- Check whether the hotspots/clustering results are meaningful/significant.





Crime events in Los Angeles

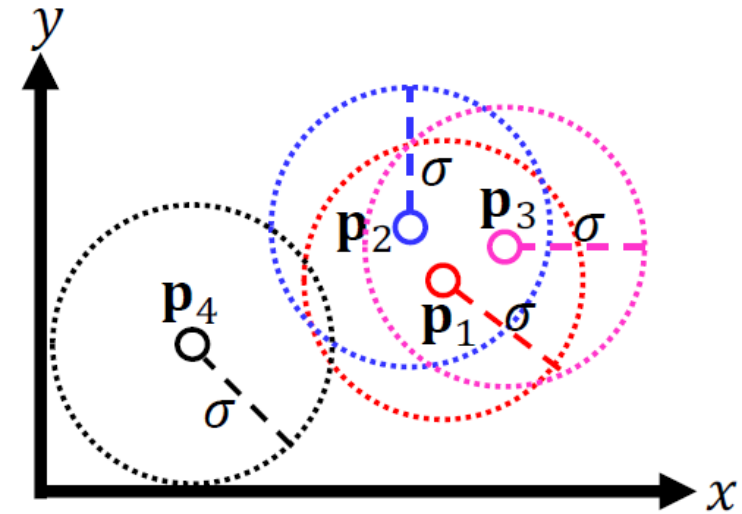


Hotspot map

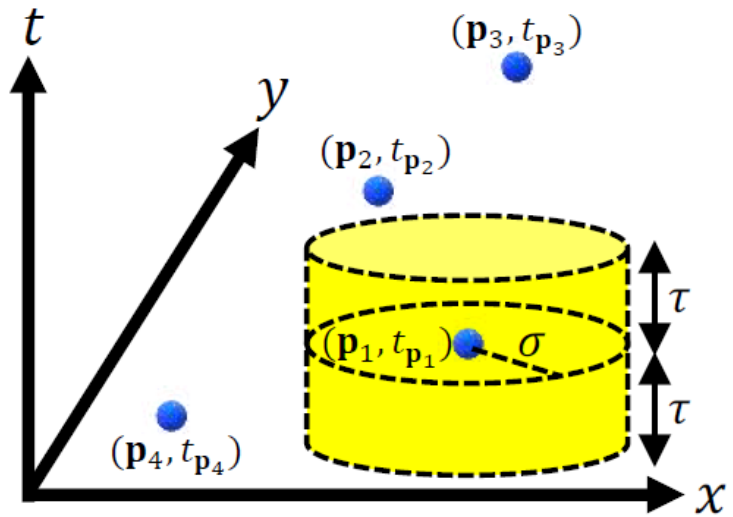
- Select suitable parameters for some clustering/hotspot detection methods.

K-function Can Be Misleading!

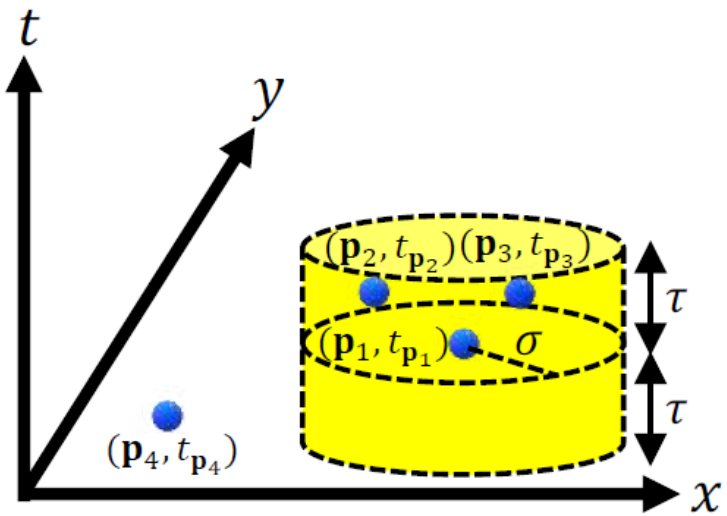
- Does not consider the temporal component of each data point. ☹



(a) Spatial distribution of data points



(b) Dispersed



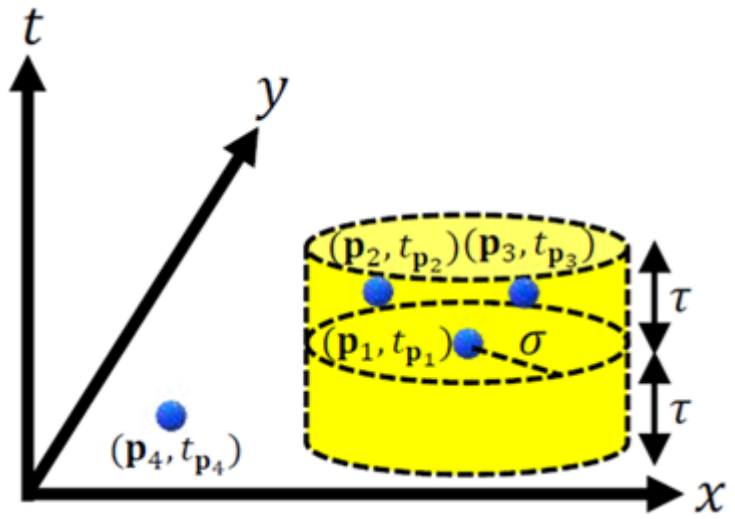
(c) Clustered

What is Spatiotemporal K-function?

- Consider a set of data points $P = \{(\mathbf{p}_1, t_{\mathbf{p}_1}), (\mathbf{p}_2, t_{\mathbf{p}_2}), \ldots, (\mathbf{p}_n, t_{\mathbf{p}_n})\}$ and the spatial threshold σ and the temporal threshold τ. The spatiotemporal K-function is:

$$K_P(\sigma, \tau) = \sum_{(\mathbf{p}_i, t_{\mathbf{p}_i}) \in P} \sum_{\substack{(\mathbf{p}_j, t_{\mathbf{p}_j}) \in P \\ j \neq i}} \mathfrak{I}\left(d(\mathbf{p}_i, \mathbf{p}_j) \leq \sigma, d\left(t_{\mathbf{p}_i}, t_{\mathbf{p}_j}\right) \leq \tau\right)$$

where d denotes the Euclidean distance.



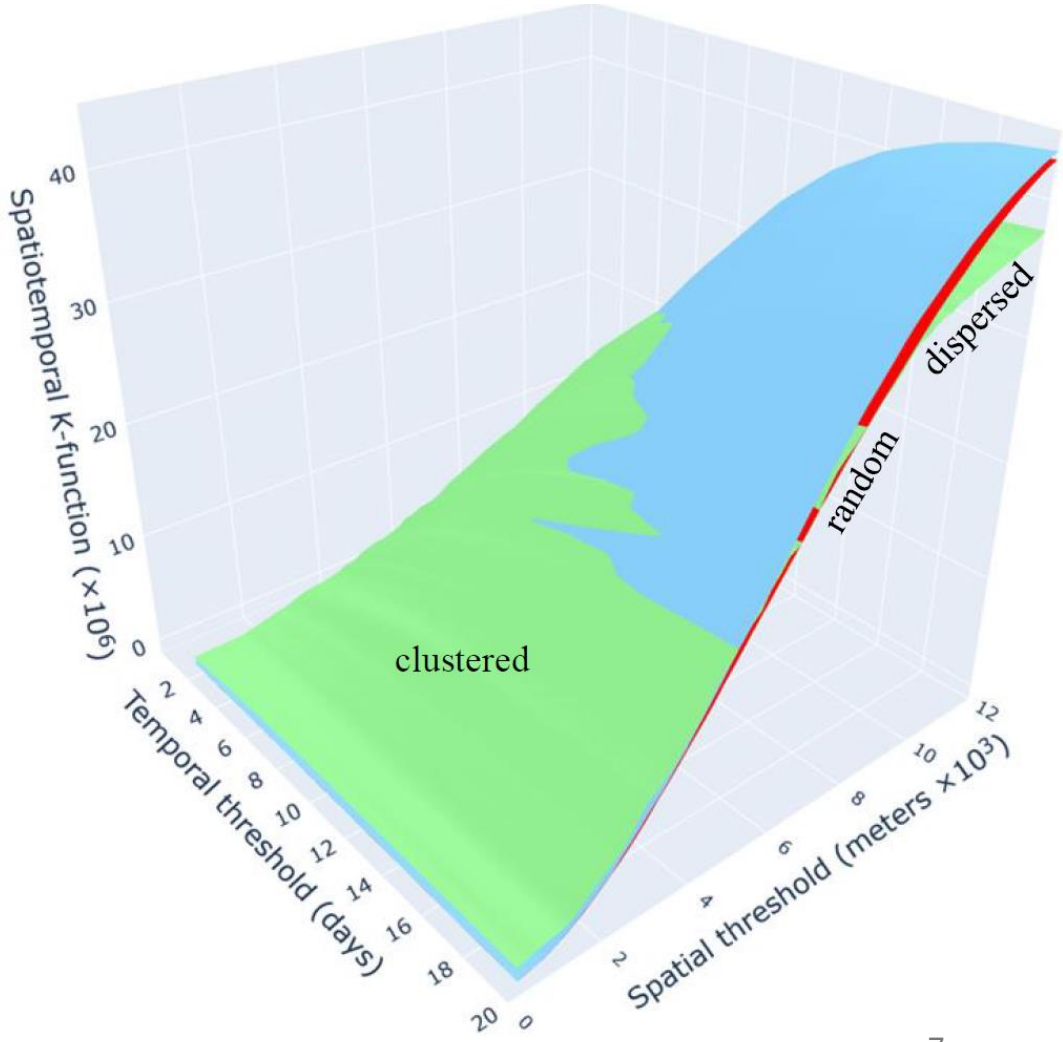
What is Spatiotemporal K-function Plot?

- Compute the spatiotemporal K-functions $K_P(\sigma, \tau)$ for the original dataset P (size n) with respect to S spatial thresholds, $\sigma_1, \sigma_2, \dots, \sigma_S$, and T temporal thresholds, $\tau_1, \tau_2, \dots, \tau_T$ (green plane).

- Generate L random datasets, $R_1, R_2, \dots, R_L$ with the same size n, and compute $\mathcal{L}(\sigma, \tau)$ and $\mathcal{U}(\sigma, \tau)$ for each (σ, τ)-pair.

$$\mathcal{L}(\sigma, \tau) = \min\left(K_{R_1}(\sigma, \tau), K_{R_2}(\sigma, \tau), \dots, K_{R_L}(\sigma, \tau)\right)$$

$$\mathcal{U}(\sigma, \tau) = \max\left(K_{R_1}(\sigma, \tau), K_{R_2}(\sigma, \tau), \dots, K_{R_L}(\sigma, \tau)\right)$$



Spatiotemporal K-function Plot is Slow!

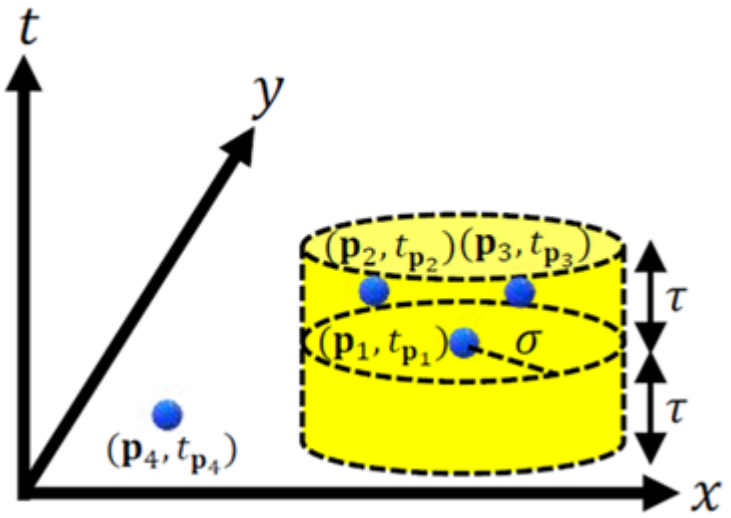
- Time complexity for generating a spatiotemporal K-function plot is $O(LSTn^2)$ ☹

- Example:
  - Number of random datasets: 4
  - Number of data points: 1,000,000
  - Number of spatial thresholds: 60
  - Number of temporal thresholds: 20
  - Total number of operations: **6000 Trillions** ☹

Range-Query-based Solution (RQS)

- Find the range query set for each data point $(\mathbf{p}_i, t_{\mathbf{p}_i})$.

$$\mathbb{R}_{\sigma,\tau}(\mathbf{p}_i, t_{\mathbf{p}_i}) = \left\{(\mathbf{p}_j, t_{\mathbf{p}_j}) \in P: \begin{matrix} d(\mathbf{p}_i, \mathbf{p}_j) \le \sigma, d\left(t_{\mathbf{p}_i}, t_{\mathbf{p}_j}\right) \le \tau \\ j \ne i \end{matrix}\right\}$$



- Compute the Spatiotemporal K-function.

$$K_P(\sigma, \tau) = \sum_{(\mathbf{p}_i, t_{\mathbf{p}_i}) \in P} \left|\mathbb{R}_{\sigma,\tau}(\mathbf{p}_i, t_{\mathbf{p}_i})\right|$$

- Many tree structures are available. ☺

- Cannot reduce the time complexity. ☹

Our Contributions

Threshold interval type	Method	Time complexity	Space complexity
Fixed threshold intervals	RQS	$O(LSTn^2)$	$O(nL + ST)$
	DNA	$O(Ln^2 + LSTn)$ (Theorem 1)	$O(nL + ST)$ (Theorem 3)
Multiple threshold intervals	RQS	$O(LSTn^2)$	$O(nL + ST)$
	DNA	$O(Ln^2 \log ST + LSTn)$ (Theorem 2)	$O(nL + ST)$ (Theorem 3)

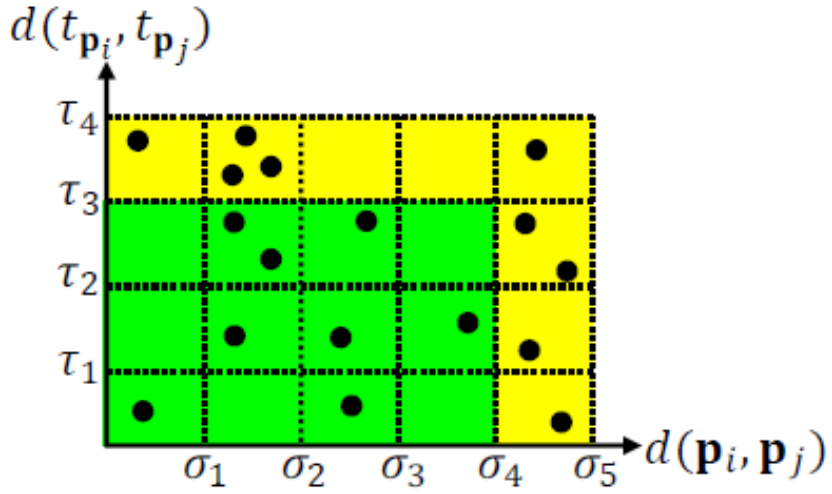
- Can reduce the worst-case time complexity for generating a spatiotemporal K-function plot, without increasing the space complexity. ☺

- Can achieve 4.58x to 57.42x speedups compared with RQS. ☺

Core Idea: Coverage Property

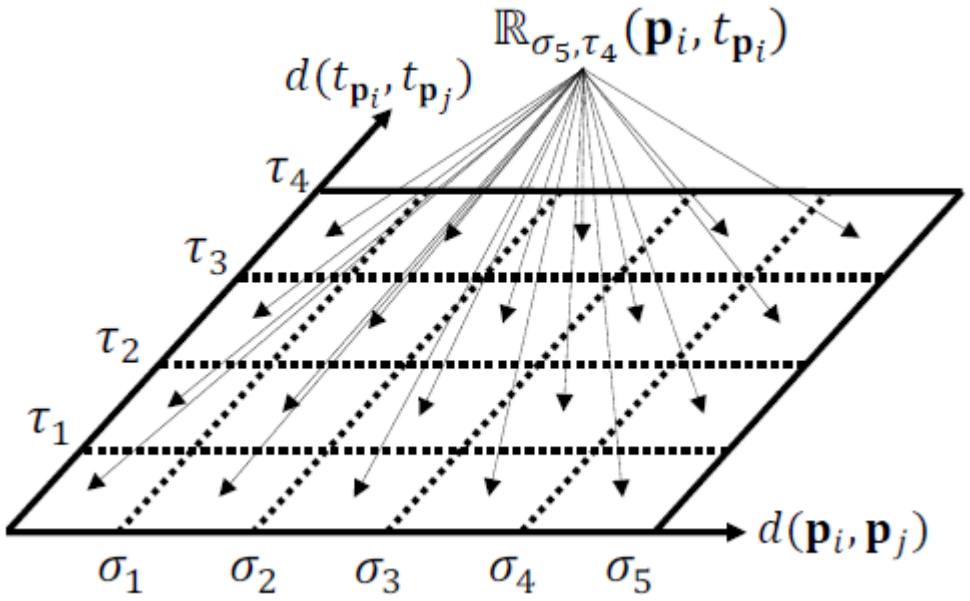
- The range query set $\mathbb{R}_{\sigma_S,\tau_T}(\mathbf{p}_i, t_{\mathbf{p}_i})$ with the largest spatial threshold σ_S and the largest temporal threshold τ_T can be shared to other range query sets $\mathbb{R}_{\sigma_u,\tau_v}(\mathbf{p}_i, t_{\mathbf{p}_i})$, where $1 \leq u \leq S$ and $1 \leq v \leq S$, i.e.,

$$\mathbb{R}_{\sigma_u,\tau_v}(\mathbf{p}_i, t_{\mathbf{p}_i}) \subseteq \mathbb{R}_{\sigma_S,\tau_T}(\mathbf{p}_i, t_{\mathbf{p}_i})$$



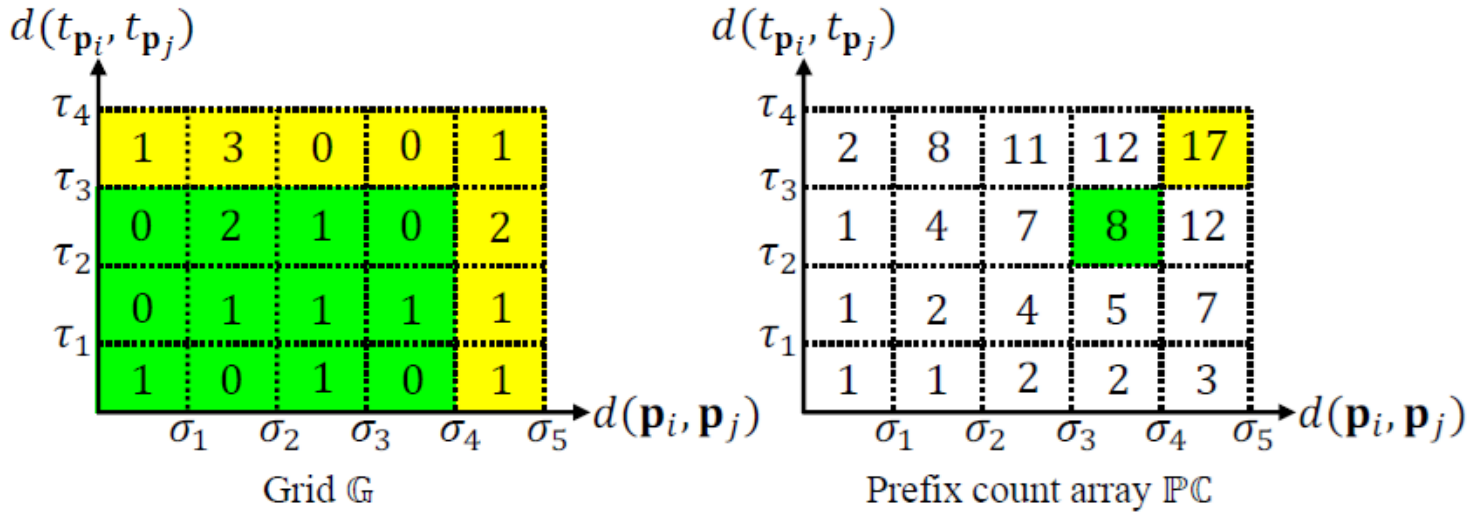
Our Solution: Distribution and Aggregation (DNA)

- Distribution



$O(n + ST)$ time

- Aggregation

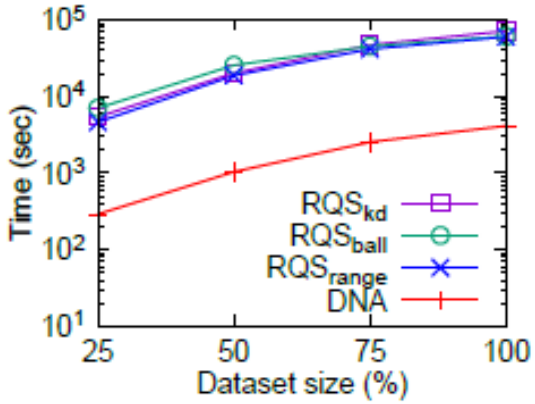


$O(ST)$ time

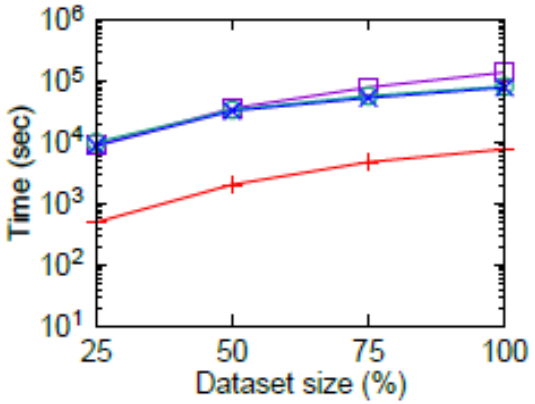
- Total time complexity is $O(Ln^2 + LSTn)$.
- Better than $O(LSTn^2)$. ☺

Experimental Evaluation

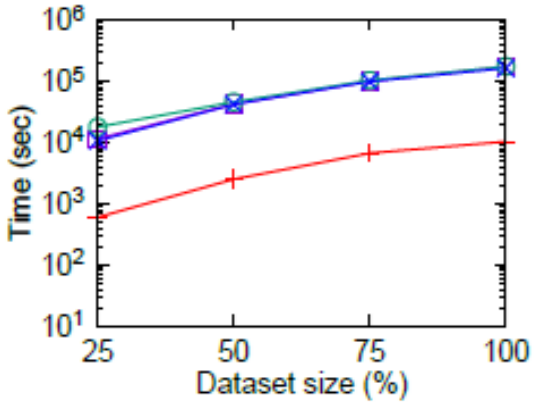
Name	n	Category
Seattle	236,136	Traffic accidents
Atlanta	363,861	Crime events
Los Angeles	559,861	Bicycle mobility
New York	1,541,284	Traffic accidents



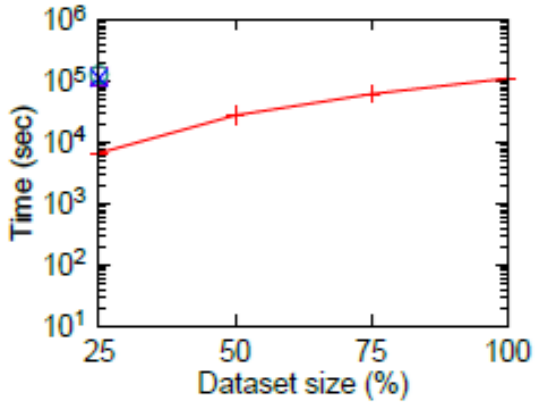
(a) Seattle



(b) Atlanta



(c) Los Angeles

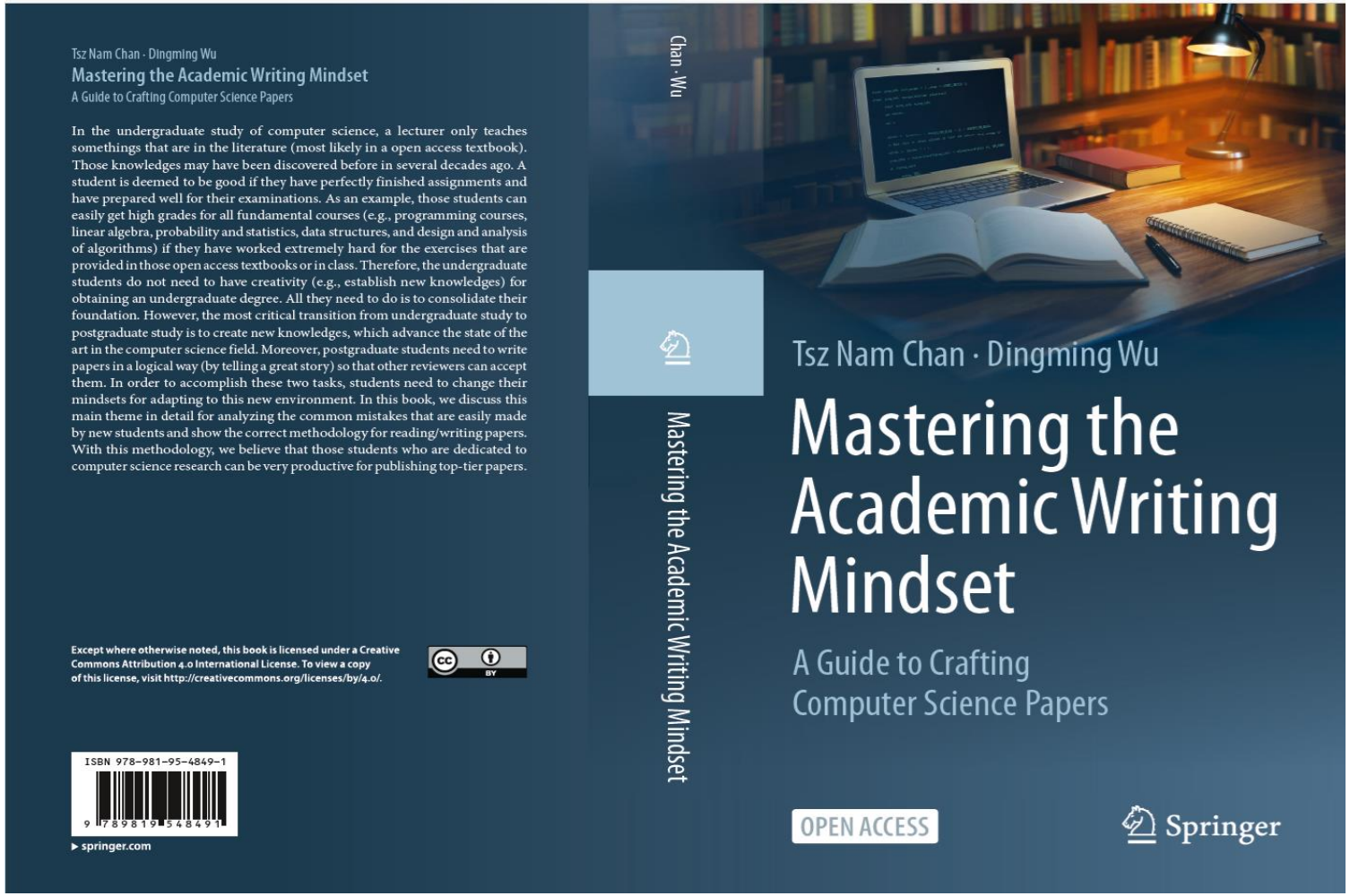


(d) New York

Future Work

- Incorporate DNA into the QGIS/ArcGIS plugin.
- Develop efficient approximate algorithms for supporting spatiotemporal K-function analysis.
- Support other GIS tools, e.g., Moran's I, Getis-Ord General G, and Kriging.
- Leverage modern hardware, e.g., GPU, to further improve the performance of DNA.

Our New Book



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